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THE CONCEPTUAL ROLE OF THE SHADOW ECONOMY IN ECONOMIC GROWTH

КОНЦЕПТУАЛЬНА РОЛЬ ТІНЬОВОЇ ЕКОНОМІКИ В ЕКОНОМІЧНОМУ ЗРОСТАННІ

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The article examines the conceptual role of the shadow and gray economies in economic growth and substantiates the need to take them into account in multifactor models of economic development. The general division of the economy into white (official), gray and shadow economies, their characteristics and peculiarities of influence on macroeconomic indicators are considered. The official economy operates within the framework of the law, provides tax revenues, promotes employment growth and investment attractiveness. The gray economy, although not illegal, partially escapes state control, which leads to tax evasion and distortion of economic processes. The shadow economy encompasses illegal activities that may contribute to the short-term survival of a business, but in the long run undermine economic stability. It is proposed to include indicators of the shadow and gray economies in multivariate economic growth models to increase the degree of adaptability of the models and to make more accurate forecasts. In economic growth models, the gray economy is taken into account by adjusting the savings parameters, while the shadow economy requires the introduction of a separate capital intensity equation based on its own production function. To correctly reflect its impact, expressions describing shadow savings and investment are added to the equation of aggregate savings.

Keywords: economic growth, models of economic growth, white (official) economy, grey (semi-legal) economy, shadow economy.

У статті досліджуються концептуальні та практичні наслідки включення тіньової та сірої економіки в моделі економічного зростання. Висвітлено обмеження традиційних моделей економічного зростання, які враховують лише офіційну (білу) економіку, що функціонує легально, забезпечує збір податкових надходжень, підтримує зайнятість та підвищує інвестиційну привабливість. У роботі розрізняють білий, сірий і тіньовий сектори: сіра економіка включає частково нерегульовану, але не обов'язково незаконну діяльність, яка призводить до фіскальних втрат і викривлень в економічних вимірах, тоді як тіньова економіка включає повністю незаконні операції, які можуть запропонувати тимчасове фінансове полегшення, але в кінцевому підсумку підривають довгострокову економічну стабільність та інституційну довіру. У дослідженні стверджується, що виключення цих неформальних сегментів призводить до значних прогалин у розробці політики та економічному прогнозуванні. У дослідженні пропонується інтегрований підхід до моделювання, в якому ефекти тіньової економіки враховуються шляхом коригування параметрів заощаджень, що відображають неформальну поведінку домогосподарств та бізнесу. Для тіньової економіки введено окреме рівняння капіталомісткості, що виводиться з унікальної виробничої функції, яка враховує специфічну динаміку нелегального використання капіталу та праці. Щоб краще відобразити вплив цих секторів, модель включає тіньові заощадження та інвестиційні компоненти в рівняння сукупних заощаджень. Такий підхід підвищує пояснювальну та прогностичну силу моделей економічного зростання, особливо в країнах з перехідною економікою та економіками, що розвиваються, де неофіційні сектори становлять значну частку загальної економічної активності. Врахування цих аспектів дозволяє розробляти більш гнучкі та точні економічні моделі, які краще відображають реальну структуру та динаміку економіки. Актуальність теми зумовлена зростанням масштабів неформальної економічної діяль-

ності в усьому світі, що суттєво впливає на макроекономічні показники, спотворює статистичні дані та ускладнює розробку ефективних економічних моделей, які адекватно відображають реальні економічні процеси.

Ключові слова: економічне зростання, моделі економічного зростання, біла (офіційна) економіка, сіра (напівлегальна) економіка, тіньова економіка.

Statement of the problem. At the present stage, economic growth modelling is becoming extremely relevant, as in the face of profound transformational changes in the global economy and growing instability, it is important to have effective and accurate tools for forecasting and analyzing economic processes. Increasing globalization, rapid technological change and other factors make economic development management more complex. Existing economic growth models based on the concept of a closed economy are becoming less and less effective. Given the shortcomings of the models, the development of new models and modification of old ones are becoming a prerequisite to better reflect current trends and provide more accurate forecasting of economic development.

Analysis of recent research and publications. There are a large number of economic growth models and their modifications in open sources. For the study [1], the main 13 that most often appear in the scientific literature were selected.

The strengths of the models include the integration of additional factors such as human capital, public capital, and land alongside the core components of labor and physical capital; the decomposition of the economy into distinct subsystems (e.g., core production and innovation sectors); the incorporation of international trade in capital dynamics; the time-dependency of indicators and factor inputs; and the modularity of the models, allowing for flexible adaptation [1].

However, several limitations remain. Many models assume a closed economy, which is incompatible with the realities of globalization and international economic interdependence. The predominance of one-dimensional models limits explanatory power compared to multidimensional approaches. Key aspects such as foreign direct investment, international aid flows, and the cyclical nature of economic activity are often omitted. Critically, the shadow economy – despite its considerable impact on national income, tax revenues, and investment flows – is largely ignored. This omission reduces the models' realism and practical applicability. Furthermore, the simplistic treatment of taxation and the absence of mathematical mechanisms to reflect its influence on public capital, as well as the neglect of market typology in supply and demand formation, weaken predictive capacity [1].

In light of these strengths and limitations, it is evident that traditional models have restricted practical utility due to substantial error margins and overly simplified assumptions. Addressing these issues through the incorporation of shadow economic dynamics and the removal of unrealistic premises will enhance the robustness of growth models. A more comprehensive, mathematically valid modelling framework tested in real-world conditions is essential for producing reliable and relevant forecasts.

Highlighting previously unresolved parts of the overall problem. The shadow and gray economies are integral components of the economic system of most countries, regardless of their level of development. They include activities that fall outside the scope of official accounting, such as informal employment, tax evasion, and the illegal production and circulation of goods and services. Although the shadow economy is often perceived as a destructive factor that reduces tax revenues and distorts market mechanisms, it can also have a positive impact, especially in conditions of instability or ineffective government regulation.

The role of the shadow sector in economic growth is controversial and multifaceted. On the one hand, the shadow economy compensates for institutional shortcomings and promotes self-employment, increasing overall employment and income levels. On the other hand, its spread can slow down structural reforms, reduce the country's investment attractiveness, and create long-term macroeconomic risks. Taking this factor into account in multifactor models of economic growth allows for a better assessment of the real drivers of economic development and forecasting its dynamics.

Formation of the objectives of the article. This article aims to analyze the role of the shadow and gray economy in the processes of economic growth, to determine the theoretical foundations of its impact and to justify the need to take this phenomenon into account in multifactor models of economic dynamics and, as a result, to form a theoretical approach to a comprehensive assessment of the economic growth of a system

that combines sectors of different levels of legality.

Summary of the main research material. The economy of any country is divided into three main segments: white (official), gray and shadow. This division reflects the level of compliance with legal norms, the degree of government control and the official accounting of economic activity. The white economy covers all legal processes regulated by law and subject to taxation. The gray economy includes activities that do not formally contravene the law, but partially or completely avoid state control, such as concealing income or paying wages in envelopes. The shadow economy, in turn, includes illegal activities such as smuggling, tax evasion, corruption schemes, and other violations of the law. Let's take a closer look at each type in more detail.

The white economy encompasses all legal economic activities that are carried out in accordance with applicable law and are subject to state regulation and accounting. It includes the production of goods and services that is recorded in official statistics, taxed, and ensures compliance with labor, environmental, and social laws. The functioning of the white economy is based on transparent market mechanisms that promote stable economic growth, job creation, investment attraction and entrepreneurship. In addition, the official economy is the main source of the state budget through the system of taxes and fees, which, in turn, allows financing social programs, infrastructure projects and other government initiatives.

The main features of the white economy are legal regulation, transparency, official accounting and taxation. Legal regulation means that all economic transactions are carried out in accordance with the established legal acts. Transparency implies open access to information about business entities, financial performance and business conditions. Official accounting is expressed in accounting and tax reporting, which allows the state and financial institutions to control economic activity. Another important aspect is taxation, which provides financial revenues to the budget and contributes to the implementation of the state social and economic policy. The combination of these features ensures the stability of the economic system, builds trust between market participants, and facilitates the country's integration into the global economy.

We will estimate the white economy using the model described in [1–3]. In the model, the main factors of production are private capital K_{pr}, public capital K_{gov}, human capital (knowledge) H, labor L and the variable factor R. Variable factor R in a single-sector production model is responsible for the land factor N. A modified Cobb-Douglas function of the form:

$$Y_p = AK_{pr}^{\alpha} K_{gov}^{\beta} H^{\gamma} N^{\varphi} L^{1-\alpha-\beta-\gamma-\varphi}, \quad (1)$$

where α – is the coefficient of elasticity of private capital, β – public capital elasticity coefficient, γ – human capital elasticity coefficient, φ – elasticity of the variable factor, in this case, land [2; 3].

In the multisectoral model, the factor R depends on the sector. For the primary sector Y_{agr} land is a factor, similar to the single-sector model. For the secondary sector Y_{ind} factor is the output of the primary sector Y_{agr}. For the tertiary sector Y_{serv} factor is the output of the secondary sector Y_{ind}.

For a multisectoral model, the production function takes the form:

$$\begin{aligned} Y_p = {} & A_1 K_{agr}^{\alpha 1} K_{gov}^{\beta 1} H_{agr}^{\gamma 1} N^{\varphi 1} L_{agr}^{1-\alpha 1-\beta 1-\gamma 1-\varphi 1} + \\ & A_2 K_{ind}^{\alpha 2} K_{gov}^{\beta 2} H_{ind}^{\gamma 2} Y_{agr}^{\varphi 2} L_{ind}^{1-\alpha 2-\beta 2-\gamma 2-\varphi 2} + \\ & A_3 K_{serv}^{\alpha 3} K_{gov}^{\beta 3} H_{serv}^{\gamma 3} Y_{ind}^{\varphi 3} L_{serv}^{1-\alpha 3-\beta 3-\gamma 3-\varphi 3}, \end{aligned} \quad (2)$$

wherein $Y_p = Y_{agr} + Y_{ind} + Y_{serv}$, similarly $K_{pr} = K_{agr} + K_{ind} + K_{serv}$ and $H = H_{agr} + H_{ind} + H_{serv}$, $L = L_{agr} + L_{ind} + L_{serv}$.

In the proposed model, capital is divided into private and public capital, which allows for a more accurate accounting of their functions and role in the process of economic growth. Investments are made through aggregate savings, which reflect the ability of the economy to effectively allocate resources for development. The dynamics of capital is characterized by three key indicators: the capital intensity of the private sector, which determines the volume of private investment; the capital intensity of the public sector, which reflects investments in public infrastructure and public goods; and aggregate savings per unit of labor, which is the main source of investment in the economy. This approach allows for a more detailed analysis of the relationship between private and public investment, as well as an assessment of their combined impact on labor efficiency and long-term economic growth [2; 3].

The innovation sector generates new knowledge by the production function:

$$\Delta H = BK_{rd}^{\upsilon} L_{rd}^{1-\upsilon}, \quad (3)$$

where K_{rd} – capital raised in the innovation sector, L_{rd} – labor involved in the innovation sector, υ – capital elasticity in the innovation sector. Total capital in the economy K_{full} can be found by the formula: $K_{full} = K_{rd} + K_{pr} + K_{gov}$, similar to labor: $L_{full} = L_{rd} + L$.

Full single-sector multivariate model in general form [2; 3]:

$$\begin{cases} k_{pr}^{"} = i_{in} + i_f - \left(d_{pr} + n\right)k_{pr}, \\ k_{gov}^{"} = g - \left(d_{gov} + n\right)k_{gov} + tx, \\ m^{"} = sAk_{pr}^{\alpha}k_{gov}^{\beta}h^{\gamma}n_N^{\varphi} - \left(g + nm + i_{in} + i_{out}\right), \\ h^{"} = Bk_{rd}^{\upsilon}l - nh, \end{cases} \quad (4)$$

where k_{pr} – capital intensity of the private sector, d_{pr} – amortization rate of private capital, n – average growth rate of the employed labor force, i_{in} – domestic investment per unit of labor, i_f – foreign investment per unit of labor, k_{gov} – capital intensity of the public sector, g – taxes per unit of labor, d_{gov} – depreciation ratio for public capital, tx – net government international transfers, m – total savings per unit of labor, s – savings rate, n_N – land factor per unit of labor, i_{out} – external investment per unit of labor.

For a multisectoral modification, the formula for the derivative of total savings per unit of labor is as follows:

$$\begin{aligned} m^{"} = {} & A_1\frac{L_{agr}}{L}k_{agr}^{\alpha 1}k_{gov}^{\beta 1}h_{agr}^{\gamma 1}n_N^{\varphi 1} + \\ & + A_2\frac{L_{ind}}{L}k_{ind}^{\alpha 2}k_{gov}^{\beta 2}h_{ind}^{\gamma 2}y_{arg}^{\varphi 2} + \\ & + A_3\frac{L_{serv}}{L}k_{serv}^{\alpha 3}k_{gov}^{\beta 3}h_{serv}^{\gamma 3}y_{ind}^{\varphi 3} - \\ & - \left(g + nm + i_{in} + i_{out}\right). \end{aligned} \quad (5)$$

The gray economy is a part of economic activity that does not directly violate the law, but escapes state control, accounting or taxation. It includes such phenomena as paying wages in envelopes, unofficial employment, underreporting of company income to minimize the tax burden, unofficial rental of real estate, or concealment of part of the revenue in small businesses. Such practices are common in the retail, construction, and service sectors, as well as among small businesses, where the costs of official activities may exceed the business's capacity.

The main features of the gray economy include incomplete or selective accounting of business transactions, partial tax and fee evasion, the use of informal employment, and insufficient transparency of financial flows. Activities in this sector are often carried out without official registration of labor relations, which allows employers to reduce the cost of social contributions. At the same time, employees may receive higher net income, but are deprived of social guarantees such as pensions or paid sick leave.

The conceptual difference between a gray and a white economy lies in the degree of legal compliance and the level of government control. In the white economy, all transactions are officially registered, taxed and comply with legal regulations, which ensures a stable economic environment. The gray economy, while remaining within the legal framework, exploits loopholes to minimize tax and regulatory costs. This creates an unequal playing field for businesses, with some companies operating officially and others underreporting their liabilities, which can lead to a distorted competitive environment and a loss of significant tax revenues for the state.

The gray economy has a twofold impact on economic growth, depending on its scale. A low level of the gray economy generally contributes to stable economic development, as businesses operate within the legal framework, which ensures market transparency, increases tax revenues, and improves investment attractiveness. At the same time, a high level of the gray economy can slow down economic growth, as it leads to lower tax revenues, increased social risks, and discrediting of official economic activity. The level of the gray economy depends on many factors, such as the effectiveness of government regulation, the level of tax burden, institutional weakness, corruption, and inefficiency of government institutions. High tax burden and complexity of business registration procedures may encourage entrepreneurs to avoid official registration, while low tax rates and ease of doing business contribute to the reduction of this sector.

Gray economy profits can increase private sector investment, but since they are not taxed, they do not increase public sector investment, which limits the government's ability to finance infrastructure projects, social programs, and other important initiatives for society. Thus, a high level of the gray economy can cause an imbalance between private investment and the need to finance public spending. This reduces the effectiveness of public policies aimed at developing social infrastructure, education, healthcare, and other areas important for sustainable economic development and the well-being of citizens. The lack of adequate public sector funding can also lead to a decline in the quality of public services, which in turn negatively affects the country's economic potential in the long run [4–8].

Since the gray economy operates on the fixed assets of the official economy, it is proposed to introduce the following equation into the model:

$$m_g = s_g A k_{pr}^{\alpha} k_{gov}^{\beta} h^{\gamma} n_N^{\varphi}, \quad (6)$$

where m_g – Savings derived from the gray sector of the economy, s_g – The rate of accumulation of the gray sector of the economy (part of the total rate of accumulation: $s_f = s_g + s$). This means that a part of the capital that could be directed to the official sector through the mechanisms of taxation and state redistribution is accumulated outside the regulated economic system. At the same time, the production function in the gray sector remains similar to the official sector, which indicates that productive characteristics are preserved, but at the same time, the absence of tax deductions from this amount limits the possibilities of public financing.

Given expression (6), we can modify the aggregate savings equation as follows:

$$m^{"} = sAk_{pr}^{\alpha} k_{gov}^{\beta} h^{\gamma} n_N^{\varphi} - \left(g + nm + i_{in} + i_{out}\right) + m_g, \quad (7)$$

or:

$$m^{"} = \left(s + s_g\right) Ak_{pr}^{\alpha} k_{gov}^{\beta} h^{\gamma} n_N^{\varphi} - \left(g + nm + i_{in} + i_{out}\right). \quad (8)$$

In general terms:

$$\begin{cases} k_{pr}^{"} = i_{in} + i_f - \left(d_{pr} + n\right) k_{pr}, \\ k_{gov}^{"} = g - \left(d_{gov} + n\right) k_{gov} + tx, \\ m^{"} = \left(s + s_g\right) Ak_{pr}^{\alpha} k_{gov}^{\beta} h^{\gamma} n_N^{\varphi} - \left(g + nm + i_{in} + i_{out}\right), \\ h^{"} = Bk_{rd}^{\upsilon} I - nh. \end{cases} \quad (9)$$

The shadow economy is a segment of economic activity that operates outside the scope of government regulation and control, including both tax evasion and mandatory reporting and completely illegal activities. It covers a wide range of phenomena, including illegal employment, smuggling, corruption, illegal production and sale of goods and services, financial fraud, money laundering, and other economic transactions that are not officially recorded. For example, unlicensed trade, production of counterfeit goods, or illegal gambling businesses are classic manifestations of the shadow economy.

The main features of the shadow economy are the complete or partial absence of official registration of economic activity, tax evasion, the use of illegal labor, and a high level of anonymity and opacity of financial transactions. Activities in this sector tend to escape state supervision, which allows businesses to reduce costs but also undermines the principles of fair competition and economic security. In addition, many shadow operations are accompanied by criminal elements, which creates additional threats to the stability of the financial system and the rule of law.

The conceptual difference between the shadow economy and the white and gray economies is its illegal status. Whereas the gray economy involves semi-legal practices that only partially fall outside the scope of state regulation, and the white economy fully complies with legal norms, the shadow sector operates in complete or overwhelming contradiction to the law. This means that the shadow economy not only deprives the state of tax revenues, but also creates significant macroeconomic and social risks, such as increased corruption, crime financing, and undermining trust in state institutions.

The shadow economy has a mixed impact on economic growth. A moderate level of the shadow sector contributes to financial stability, good governance, and an attractive investment environment, while its excessive spread undermines the foundations of economic development. With a small share of the shadow economy, businesses operate within the law, which ensures stable tax revenues, transparency of financial flows, and fair competition. At the same time, excessive growth of the shadow sector leads to a loss of control over financial resources, increased corruption, capital outflows, and a decrease in investment attractiveness. The level of the shadow economy is determined by a combination of factors, such as the tax and administrative burden, the effectiveness of law enforcement agencies, the level of corruption, trust in government institutions, and the overall level of economic development. In countries with high taxes, complex bureaucracy and weak governance, the shadow economy tends to grow as businesses and individuals seek to avoid excessive regulatory restrictions [4–8].

Let's assume that the shadow economy operates separately from other types and follows a similar production function:

$$Y_{sh} = CK_{sh}^{\alpha_s} K_{gov}^{\beta_s} H^{\gamma_s} N^{\varphi_s} L^{1-\alpha_s-\beta_s-\gamma_s-\varphi_s}, \quad (10)$$

where Y_{sh} – output of the shadow economy, K_{sh} – capital of the shadow economy, C – coefficient of technological progress, $\alpha_s, \beta_s, \gamma_s, \varphi_s$ – parameters of the production function, elasticity coefficients of shadow capital, public capital, human capital, and land, respectively. Since the shadow economy operates in the same system as the official one, the factors of public capital, human capital, labor, and land as a resource will be similar to those for the white sector. For example, shadow sector counterparties use the same infrastructure as official sector counterparties.

Thus, the institutional, legal, and economic conditions that determine the productivity of

the formal economy also affect the informal economy. For example, the level of education and skills of workers determines productivity not only in the formal sector, but also in the informal sector. Similarly, the quality of transportation, financial, and communication infrastructure contributes to the efficiency of both legal and illegal businesses. Moreover, the shadow sector can adapt to the constraints and incentives imposed by the state on the formal economy. For example, a high tax burden or overregulation of business activities can encourage the flow of resources to the informal sector. At the same time, a well-developed legal system and effective control can reduce the size of the shadow economy or force it to operate under conditions of increased transaction costs.

The dynamics of the shadow sector will be displayed similarly:

$$k_{sh}^{"} = i_{sh} - \left(d_{sh} + n\right)k_{sh}, \qquad (11)$$

where i_{sh} – investments in the shadow sector, d_{sh} – depreciation ratio of shadow capital.

Taking into account the additional equation to the model, we get:

$$\begin{cases} k_{pr}^{\bullet} = i_{in} + i_f - \left(d_{pr} + n\right)k_{pr}, \\ k_{gov}^{\bullet} = g - \left(d_{gov} + n\right)k_{gov} + tx, \\ k_{sh}^{\bullet} = i_{sh} - (d_{sh} + n)k_{sh} \\ m^{\bullet} = \left(s + s_g\right)Ak_{pr}^{\alpha}k_{gov}^{\beta}h^{\gamma}n_N^{\varphi} + s_{sh}Ck_{sh}^{\alpha_s}k_{gov}^{\beta_s}h^{\gamma_s}n_N^{\varphi_s} - \\ -(g + nm + i_{in} + i_{out} + i_{sh}), \\ h^{\bullet} = Bk_{rd}^{v}l - nh. \end{cases} \qquad (12)$$

where s_{sh} – rate of accumulation for the shadow sector.

The accumulation of capital in the shadow sector directly affects the formation of aggregate savings in the economy, since, despite the unofficial nature of shadow transactions, they remain part of economic circulation. Shadow financial flows can accumulate significant amounts of resources, which are subsequently integrated into the overall system of national savings and investment process.

In the structure of the economy's aggregate savings, financial resources are further divided into three main areas: official domestic investment, which is directed to the development of legal businesses and government projects; domestic shadow investment, which is used for illegal or semi-legal economic activities; and official foreign investment, which is realized through investments in foreign assets or international financial markets.

Conclusions. The shadow and gray economies are important factors that affect economic growth, but their effects on the economy differ. The gray economy encompasses semi-legal practices that do not directly contradict the law but partially avoid state control and taxation. It can stimulate private sector development, but it also reduces tax revenues and creates unequal conditions for business. The shadow economy, unlike the gray economy, includes completely illegal activities that not only deprive the state of financial resources, but also promote corruption, criminalize economic processes and undermine trust in state institutions.

The level of gray and shadow economy has a significant impact on economic development. A moderate level of the gray economy can partially compensate for the shortcomings of government regulation, but its excessive development weakens public finances and social policy. At the same time, a high level of the shadow economy is a critical threat to sustainable development, as it leads to a loss of control over financial flows and the spread of illegal practices. The factors that determine the extent of these phenomena include tax policy, the level of bureaucracy, the quality of public administration, and the degree of corruption.

In mathematical models of economic growth, the gray economy can be integrated as an additional parameter in the savings function, as it retains the main characteristics of the formal sector, except for the full tax burden and government regulation. This allows its impact to be taken into account in the overall savings rate by adjusting the savings parameters accordingly.

The shadow economy, on the other hand, is characterized by a more complex structural logic, as it operates separately from the official sector and often has illegal sources of financing. To account for it correctly in economic growth models, it is necessary to introduce a separate equation describing shadow capital intensity, which is based on an independent production process. In addition, the equation of aggregate savings should include additional expressions reflecting shadow savings and investments that operate outside the official economic circulation. This approach allows us to adequately model the impact of the shadow sector on macroeconomic dynamics and assess its consequences for overall economic development.

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