MODELLING OF PRICING OF A CONVERTED BOND

Yanishevskyi V.S.
Candidate of Physico-Mathematical Sciences, Associate Professor,
Senior Lecturer at Department of Management Technologies,
Lviv Polytechnic National University

A two-factor pricing model of convertible bonds is investigated. The model is based on Black-Scholes idea of a risk-free portfolio, which is based on bonds and underlying asset. A dynamic of the price is given by the stochastic equation of geometric Brownian motion; the Vasicek model is used for interest rates. Using the methods of stochastic analysis, the differential equation in partial derivatives for bond price dynamics is obtained. The equation was investigated in the literature using a direct method. Herewith by means of transformations, the equation was reduced to an ordinary differential equation, for which the solution was found. However, the solution found was a cumbersome expression and the resulted formula for the price of the converted bond was not provided.

In the paper, an analogy method is applied to the analysis of differential equation of bond price. It is in transforming of the differential equation of bond price to known models of mathematical physics. As the result of Fourier transformation by x variable (x=ln(S), S – stock price), the order of differential equation is reduced. The resulting equation contains coefficients, which depends on the Fourier variable k, as a parameter. As the result of the subsequent transformation made, it is shown that the indicated equation is equal to an equation of quantum mechanics, which describes an oscillator-like system. The oscillator model is well known and researched. This allowed using well-known results of quantum mechanics; Green's function of oscillator equation in particular. By doing reverse transform, we received a Green function of the initial dynamics equation of bond price, and we stated its solution depending on initial conditions.

To summarize, a formula of the price of the converted bond of European-style is obtained. It is also shown that if to choose the initial condition independent of bond price then we receive the income formula in Vasicek interest rate model for the ordinary bond. Received equations are written in the form, which is practical for application. For further application of the received formulae, their calibration is needed based on statistic data of bond markets.